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Perception-Based Image Classification

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Abstract

Purpose - The purpose of this paper is to present near set theory using the perceptual indiscernibility and tolerance relations, to demonstrate the practical application of near set theory to the image correspondence problem, and to compare this method with existing image similarity measures.
Design/Methodology/Approach - Image correspondence methodologies are present in many systems that we depend on daily. In these systems, the discovery of sets of similar objects (*aka*, tolerance classes) stems from human perception of the objects being classified. This view of perception of image correspondence springs directly from J.H. Poincaré's work on visual spaces during the 1890s and E.C. Zeeman's work on tolerance spaces and visual acuity during 1960s. Thus, in solving the image correspondence problem, it is important to have systems that accurately model human perception. Near set theory provides a framework for measuring the similarity of digital images (and perceptual objects, in general) based on features that describe them in much the same way that humans perceive objects.
Findings - The contribution of this article is a perception-based classification of images using near sets.
Originality/value - The method presented in this paper represents a new approach to solving problems in which the goal is to match human perceptual groupings. While the results presented in the paper are based on measuring the resemblance between images, the approach presented in this article can be applied to any application that can be formulated in terms of sets such that the objects in the sets can be described by feature vectors.

Keywords: Feature values; Hausdorff distance; image correspondence; perceptual tolerance relation; near sets; perception; probe function; similarity.

1. Introduction

The problem addressed in this article is one of reconciling human perception with that of image processing and image correspondence systems. The term *perception* appears in the literature in many different places with respect to the processing of images. For instance, the term is often used for demonstrating that the performance of methods are similar to results obtained by human subjects (as in [Montag and Fairchild (1997)]), or it is used when the system is trained from data generated by human subjects (as in [El-Naqa *et al.* (2004)]). Thus, in these examples, a system is considered perceptual if it mimics human behaviour. Another illustration of the use of perception is in the area of semantics with respect to queries [Rahman *et al.* (2007); Martinez *et al.* (2005)]. For instance, [Martinez *et al.* (2005)] focuses on queries for 3-D environments, *i.e.*, performing searches of an online virtual environment. Here the question of perception is one of semantics and conceptualization with regard to language and queries. For example, a user might want to search for the tall tree they remembered seeing on one of their visits to a virtual city.

Other interpretations of *perception* are tightly coupled to psychophysics introduced in 1860 in [Fechner (1966, 1860)], *i.e.*, perception based on the relationship between stimuli and sensation. For example, [Papathomas *et al.* (1997)] introduces a texture perception model. The texture perception model uses the antagonistic view of the Human Visual System (HVS) in which our brain processes differences in signals received from rods and cones rather than sense signals, directly. An image-feature model of perception has been suggested by Mojsilovic *et al.* [2002], where it is suggested that humans view/recall an image by its dominant colours only, and areas containing small, non-dominant colours are averaged by the HVS. Other examples of the term perception defined in the context of psychophysics have also been given [Balakrishnan *et al.* (2005); Qamra *et al.* (2005); Wang *et al.* (2004); Dempere-Marco *et al.* (2002); Kuo and Johnson (2002); Wandell *et al.* (2002); Wilson *et al.* (1997)].

Perception as explained by psychologists [Hoogs *et al.* (2003); Bourbakis (2002)] is similar to the understanding of perception in psychophysics. In a psychologist's view of perception, the focus is more on the mental processes involved rather than interpreting external stimuli. For example, [Bourbakis (2002)] presents an algorithm for detecting the differences between two images based on the representation of the image in the human mind (*e.g.*, colours, shapes, and sizes of regions and objects) rather than on interpreting the stimuli produced when looking at an image. In other words, the stimuli from two images have been perceived and the mind must now determine the degree of similarity.

The view of perception presented in this article combines the basic understanding of perception in psychophysics with a view of perception found in Merleau-Ponty's work [1945]. That is, perception of an object (*i.e.*, in effect, our knowledge about an object) depends on information gathered by our senses. Perception of objects is identified with our ability to describe what we perceive. In other words, object perception is synonymous with object description. The proposed approach to perception is feature-based and is similar to the one discussed in the introduction of [Calitoiu *et al.* (2007)]. In this view, our senses are likened to probe functions (*i.e.*, mappings of sensations to values assimilated by the mind). A human sense modelled as a probe measures the physical characteristics of objects in our environment. The sensed physical characteristics of an object are identified with object features.

It is our mind that identifies relationships between object feature values to form perceptions of sensed objects [Merleau-Ponty (1945)]. In this article, we show that perception, *i.e.* human perception, can be quantified through the use of near sets by providing a framework for comparing objects based on object descriptions. Objects that have similar appearance (*i.e.*, objects with similar descriptions) are considered *perceptually near each other*. Sets are considered near each other when they have “things” (perceived objects) in common. Specifically, near sets facilitate measurement of similarity between objects based on feature values (obtained by probe functions) that describe the objects. This approach is similar to the way human perceive objects (see, *e.g*, [Fahle and Poggio (2002)]) and as such facilitates pattern classification systems.

Much work has been reported in the area of near sets [Peters (2007a,b); Peters *et al.* (2007a); Henry and Peters (2009a); Peters and Wasilewski (2009); Pal and Peters (2010)], which are an outgrowth of the rough set approach to obtaining approximate knowledge of objects that are known imprecisely [Pawlak (1981, 1982); Pawlak and Skowron (2007c,b,a)]. In particular, this article presents a practical application of near set theory to the image correspondence problem and Content-Based Image Retrieval (CBIR). The growth of CBIR in the early 90’s as a research area can be attributed to increased access to capturing and storing digital images, as well as the advent of the internet as a way to share images [Smeulders *et al.* (2000)]. Since then CBIR has become a major research area with applications ranging to common everyday use, such as searching a database of personal images, to specialized content-based medical image retrieval systems [Guldogan (2008)]. CBIR systems consist of two major components, namely, similarity measures for assessing the similarity of images, and indexing to perform fast comparison and retrieval of images from a database. The problem presented in this article is an introduction to a similarity measure based on near set theory. Particularly, a Nearness Measure (NM) for quantifying the similarity of near sets is used to perform Content-Based Image Retrieval (CBIR), and the results are compared using a traditional Hausdorff distance [Hausdorff (1914, 1962)] as well as Perceptually Modified Hausdorff Distance (PMHD) [Park *et al.* (2008)].

This article is organized as follows: Section 2 gives a brief introduction to near sets with an emphasis on indiscernibility and tolerance relations. Section 3 outlines the steps for combining near set theory with image processing for use in image retrieval. A perceptual tolerance relation useful in discerning resemblances between images is given in Section 2.1. An overview of near sets and image correspondence is given in Section 3. A brief introduction to perceptual image processing is given in Section 3.1, followed by examples of near images in Section 3.2. Then Section 4 provides an overview of the original Hausdorff distance measure along with the more recent perceptually modified Hausdorff measure. Both forms of Hausdorff measures provide a basis for a thorough comparison with a perceptual nearness measure considered in the context of tolerance spaces. Section 5 presents a comparison of results using using near sets and both forms Hausdorff distance for content-based image retrieval (CBIR). The work presented in this article is a continuation of recent applications of near set theory reported in [Henry and Peters (2007, 2008); Hassanien *et al.* (2009); Peters *et al.* (2007b); Peters and Ramanna (2007); Meghdadi *et al.* (2009); Peters and Puzio (2009); Ramanna (2010); Pal and Peters (2010)], and the contribution of this work is a step toward perception-based image retrieval.

2. Near Sets

Near sets are disjoint sets that resemble each other [Henry and Peters (2009b)]. Resemblance between disjoint sets occurs whenever there are observable similarities between the objects in the sets. Similarity is determined by comparing lists of object feature values. Each list of feature values defines an object's description. Comparison of object descriptions provides a basis for determining the extent that disjoint sets resemble each other. Objects that are perceived as similar based on their descriptions are grouped together. These groups of similar objects can provide information and reveal patterns about objects of interest in the disjoint sets.

Near set theory focuses on sets of perceptual objects with similar descriptions. Specifically, let O represent a set of perceptual objects (*i.e.* objects that have their origin in the physical world), and let $\mathcal{B}$ denote a set of real-valued functions, called probe functions, representing object features, and let $\phi_i \in \mathcal{B}, \phi_i : O \rightarrow \mathbb{R}$. The description of an object $x \in O$ is a vector given by

$$\boldsymbol{\phi}_{\mathcal{B}}(x) = (\phi_1(x), \phi_2(x), \ldots, \phi_i(x), \ldots, \phi_l(x)),$$

where l is the length of the description and each $\phi_i(x)$ is a probe function representing a feature value of x. Furthermore, we can define a set $\mathbb{F}$ (such that $\mathcal{B} \subseteq \mathbb{F}$) representing all the probe functions used to describe an object x. Next, a perceptual information system S can be defined as $S = \langle O, \mathbb{F}, \{Val_{\phi_i}\}_{\phi_i \in \mathbb{F}} \rangle$, where $\mathbb{F}$ is the set of all possible probe functions that take as the domain objects in O, and $\{Val_{\phi_i}\}_{\phi_i \in \mathbb{F}}$ is the value range of a function $\phi_i \in \mathbb{F}$. For simplicity, a perceptual system is abbreviated as $\langle O, \mathbb{F} \rangle$ when the range of the probe functions is understood. It is the notion of a perceptual system that is at the heart of the following definitions.

Definition 1. ***Normative Indiscernibility Relation*** *[Peters (2007c)] Let* $\langle O, \mathbb{F} \rangle$ *be a perceptual system. For every* $\mathcal{B} \subseteq \mathbb{F}$ *the normative indiscernibility relation* $\sim_{\mathcal{B}}$ *is defined as follows:*

$$\sim_{\mathcal{B}} = \{(x, y) \in O \times O : \parallel \boldsymbol{\phi}_{\mathcal{B}}(x) - \boldsymbol{\phi}_{\mathcal{B}}(y) \parallel_2 = 0\},$$

where $\|\cdot\|_2$ *represents the* l^2 *norm. If* $\mathcal{B} = \{\phi\}$ *for some* $\phi \in \mathbb{F}$*, instead of* $\sim_{\{\phi\}}$ *we write* $\sim_\phi$*.*

Defn. 1 is a refinement of the original indiscernibility relation given by Pawlak [1981]. Using the indiscernibility relation, objects with matching descriptions can be grouped together forming granules of highest object resolution determined by the probe functions in $\mathcal{B}$. This gives rise to an elementary set (also called an equivalence class)

$$x_{/\sim_{\mathcal{B}}} = \{x' \in X \mid x' \sim_{\mathcal{B}} x\}, \tag{1}$$

defined as a set where all objects have the same description. Similarly, a quotient set is the set of all elementary sets defined as

$$O_{/\sim_{\mathcal{B}}} = \{x_{/\sim_{\mathcal{B}}} \mid x \in O\}.$$

Defn. 1 provides the framework for comparisons of sets of objects by introducing a concept of nearness within a perceptual system. Sets can be considered near each other when they have "things" in common. In the context of near sets, the "things" can be quantified by granules of a perceptual system, *i.e.*, the elementary sets. The simplest example of nearness between sets sharing "things" in common is the case when two sets have indiscernible elements. This idea leads to the definition of a weak nearness relation.

Definition 2. ***Weak Nearness Relation*** *[Henry and Peters (2008)]*
Let $\langle O, \mathbb{F}\rangle$ *be a perceptual system and let* $X, Y \subseteq O$*. A set* X *is weakly near to a set* Y *within the perceptual system* $\langle O, \mathbb{F}\rangle$ $(X \underline{\bowtie}_{\mathbb{F}} Y)$ *iff there are* $x \in X$ *and* $y \in Y$ *and there is* $\mathcal{B} \subseteq \mathbb{F}$ *such that* $x \sim_{\mathcal{B}} y$*. In the case where sets* X, Y *are defined within the context of a perceptual system as in Defn 2, then* X, Y *are weakly near each other.*

An example of Defn. 2 is given in Fig. 1 where the straight lines represent equivalence classes. The sets X and Y are weakly near each other in Fig. 1 because they both share objects belonging to the same equivalence class.

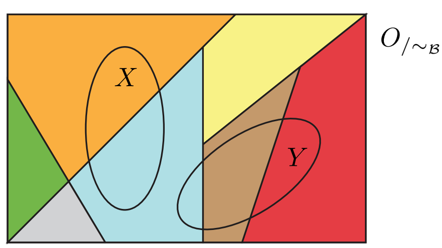


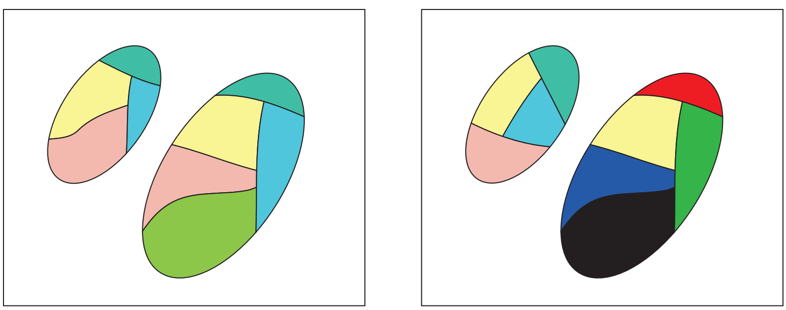
Fig. 1. Example of Defn. 2.

Defn. 2 can be used to define a Nearness Measure (NM) [Hassanien *et al.* (2009)]. Let X and Y be two weekly near (using Defn. 2) disjoint sets, and let $Z = X \cup Y$. Then, a NM between X and Y is given by

$$tNM_{\cong_{\mathcal{B}}}(X,Y) = \frac{1}{|Z_{/\sim_{\mathcal{B}}}|} \cdot \sum_{C \in Z_{/\sim_{\mathcal{B}}}} |C| \frac{\min(|C \cap X|, |[C \cap Y|)}{\max(|C \cap X|, |C \cap Y|)}. \tag{2}$$

The idea behind Eq. 2 is that sets that are similar should have similar number of objects in each equivalence class. Thus, for each equivalence class obtained from $Z = X \cup Y$, Eq. 2 counts the number of objects that belong to X and Y and takes the ratio (as a proper fraction) of their cardinalities. Furthermore, each ratio is weighted by the total size of the equivalence class (thus giving importance to the larger classes) and the final result is normalized by dividing by the sum of all the cardinalities. The range of Eq. 2 is in the interval [0,1], where a value of 1 is obtained if the sets are equivalent and a value of 0 is obtained if they have no elements in common.

As an example of the degree of nearness between two sets, consider Fig. 2 in which each image consists of two sets of objects, X and Y. Each colour in the figures corresponds to an elementary set where all the objects in the class share the same description. The idea behind Eq. 2 is that the nearness of sets in a perceptual system is based on the cardinality of equivalence classes that they share. Thus, the sets in Fig. 2(a) are closer (more near) to each other in terms of their descriptions than the sets in Fig. 2(b).



(a) (b)

Fig. 2. Example of degree of nearness between two sets: (a) High degree of nearness, and (b) low degree of nearness.

2.1. *Perceptual Tolerance Relation*

A perception-based approach to discovering resemblances between images leads to a tolerance class form of near sets that models human perception in a physical continuum viewed in the context of image tolerance spaces. A tolerance space-based approach to perceiving image resemblances hearkens back to the observation about perception made by Ewa Orłowska [1982] (see, also, [Orłowska (1985)]), *i.e.*, classes defined in an approximation space serve as a formal counterpart of perception.

The term *tolerance space* was coined by E.C. Zeeman [1962], elaborated in[Zeeman and Buneman (1968)], in modelling visual perception with tolerances. A tolerance space is a set X supplied with a binary relation $\cong$ (*i.e.*, a subset $\cong\ \subset X \times X$) that is reflexive (for all $x \in X$, $x \cong x$) and symmetric (*i.e.*, for all $x, y \in X$, $x \cong y$ implies $y \cong x$) but transitivity of $\cong$ is not required. The basic idea is to find objects that resemble each other with a tolerable level of error. Sossinsky [1986] observes that main idea underlying tolerance theory comes from J.H. Poincaré, who introduced representative spaces (*aka*, tolerances spaces) [Poincaré (1895)],elaborated in [Poincaré (1902); Poincaré (1913)]. Physical continua (*e.g.*, measurable magnitudes in the physical world of medical imaging [Hassanien *et al.* (2009)]) are contrasted with the mathematical continua (real numbers) where almost solutions are common and a given equation has no exact solutions. An *almost solution* of an equation (or a system of equations) is an object which, when substituted into the equation, transforms it into a numerical 'almost identity', *i.e.*, a relation between numbers which is true only approximately (within a prescribed tolerance) [Sossinsky (1986)]. Equality in the physical world is meaningless, since it can never be verified either in practise or in theory. Hence, the basic idea in a tolerance space view of the world, for example, is to replace the indiscernibility relation in rough sets [Pawlak (1982)] with a tolerance relation in partitioning sets into homologous regions where there is a high likelihood of overlaps, *i.e.*, non-empty intersections between tolerance classes. The use of tolerance spaces in this work is directly related to recent work on tolerance spaces (see, *e.g.*, [Hassanien *et al.* (2009); Peters (2009a,b, 2010); Peters and Ramanna (2009); Gerasin *et al.* (2008); Zheng *et al.* (2005); Bartol *et al.* (2004); Skowron and Stepaniuk (1996); Schroeder and Wright (1992); Shreider (1970); Pal and Peters (2010)]).

When dealing with perceptual objects (especially, components in images), it is sometimes necessary to relax the equivalence condition of Defn. 1 to facilitate observation of associations in a perceptual system. This variation is called a tolerance relation that defines yet another form of near sets [Peters and Ramanna (2009); Peters (2009a,b)] and is given in Defn. 3.

Definition 3. ***Perceptual Tolerance Relation*** *[Peters and Ramanna (2009)]* *Let $\langle O, \mathbb{F} \rangle$ be a perceptual system and let $\varepsilon \in \mathbb{R}$. For every $\mathcal{B} \subseteq \mathbb{F}$ a reflexive and symmetric tolerance relation is defined as follows:*

$$\cong_{\mathcal{B},\epsilon} = \{(x, y) \in O \times O : \parallel \boldsymbol{\phi}_{\mathcal{B}}(x) - \boldsymbol{\phi}_{\mathcal{B}}(y) \parallel_2 \leq \varepsilon\}.$$

If $\mathcal{B} = \{\phi\}$ for some $\phi \in \mathbb{F}$, instead of $\cong_{\{\phi\}}$ we write $\cong_{\phi}$. Further, for notational convenience, we will write $\cong_{\mathcal{B}}$ instead of $\cong_{\mathcal{B},\varepsilon}$ with the understanding that ε is inherent to the definition of the tolerance relation.

A set $X \subseteq O$ is a pre-class when $x \cong_{\mathcal{B}} y$ for any pair $x, y \in X$ [Schroeder and Wright (1992)]. A maximal pre-class with respect to inclusion is called a tolerance class. A maximal pre-class is the akin to the concept of an elementary set when using the tolerance relation instead of the indiscernibility relation, and the set $H^{\varepsilon}_{\mathcal{B}}(O)$ denoting the family of all tolerance classes of relation $\cong_{\mathcal{B}}$ on O is similar to a quotient set. However, $H^{\varepsilon}_{\mathcal{B}}(O)$ is a covering of O rather than the partition of O given by $O_{/\sim_{\mathcal{B}}}$. Finally, notice that the tolerance relation is a generalization of the indiscernibility relation given in Defn. 1 (obtained by setting $\varepsilon = 0$). As a result, Defn. 2 and Eq. 2 can be redefined with respect to the tolerance relation[a].

The following simple example highlights the need for a tolerance relation as well as demonstrates the construction of tolerance classes from real data. Consider the 20 objects in Table 1 that where $|\phi(x_i)| = 1$. Letting $\varepsilon = 0.1$ gives the following tolerance classes:

$$\begin{aligned} X_{/\cong_{\mathcal{B}}} = \{ & \{x_1, x_8, x_{10}, x_{11}\}, \{x_1, x_9, x_{10}, x_{11}, x_{14}\}, \\ & \{x_2, x_7, x_{18}, x_{19}\}, \\ & \{x_3, x_{12}, x_{17}\}, \\ & \{x_4, x_{13}, x_{20}\}, \{x_4, x_{18}\}, \\ & \{x_5, x_6, x_{15}, x_{16}\}, \{x_5, x_6, x_{15}, x_{20}\}, \\ & \{x_6, x_{13}, x_{20}\}\} \end{aligned}$$

Observe that each object in a tolerance class satisfies the condition $\| \boldsymbol{\phi}(x) - \boldsymbol{\phi}(y) \| \leq \epsilon$, and that almost all of the objects appear in more than one class. Moreover, there would be twenty classes if the indiscernibility relation was used since there are no two objects with matching descriptions.

Table 1. Tolerance Class Example.

x_i	$\phi(x)$	x_i	$\phi(x)$	x_i	$\phi(x)$	x_i	$\phi(x)$
x_1	.4518	x_6	.6943	x_{11}	.4002	x_{16}	.6079
x_2	.9166	x_7	.9246	x_{12}	.1910	x_{17}	.1869
x_3	.1398	x_8	.3537	x_{13}	.7476	x_{18}	.8489
x_4	.7972	x_9	.4722	x_{14}	.4990	x_{19}	.9170
x_5	.6281	x_{10}	.4523	x_{15}	.6289	x_{20}	.7143

3. Near Sets and Image Correspondence

Near set theory can be used to determine the nearness between two images. The following sections describe an approach for applying near set theory to images and then demonstrates it uses at measuring the similarity between images.

[a]The two relations were treated separately in the interest of clarity.

3.1. *Perceptual Image Processing*

Near set theory can be easily applied to images. For example, define a RGB image as $f = \{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_T\}$, where $\mathbf{p}_i = (c, r, R, G, B)^{\mathrm{T}}$, $c \in [1, M]$, $r \in [1, N]$, $R, G, B \in [0, 255]$, and M, N respectively denote the width and height of the image and $M \times N = T$. Further, define a square subimage as $f_i \subset f$ with the following conditions:

$$\begin{aligned} &f_1 \cap f_2 \ldots \cap f_s = \emptyset, \\ &f_1 \cup f_2 \ldots \cup f_s = f, \end{aligned} \tag{3}$$

where s is the number of subimages in f. The approach taken in this paper is to restrict all subimages to be square except when doing so violates Eq. 3. For example, the images in the Berkeley Segmentation Dataset [Martin *et al.* (2001)] often have the dimension 321×481. Consequently, a square subimage size of 25 will produce 6240 square subimages, 96 subimages of size 1×5, 64 subimages of size 5×1 and 1 subimage consisting of a single pixel. Next, O can be defined as the set of all subimages, *i.e.*, $O = \{f_1, \ldots, f_s\}$, and $\mathbb{F}$ is a set of functions that operate on images (see, *e.g.* [Henry and Peters (2009c); Marti *et al.* (2001)] for examples of probe functions). Once the set $\mathcal{B}$ has been selected, the elementary sets or tolerance classes are simply created based on the feature vector associated with each subimage.

3.2. *Example of near images*

The nearness of two images can be discovered by partitioning each one into subimages and letting these represent objects in a perceptual system, *i.e*, let the sets X and Y represent the two images to be compared where each set consists of the subimages obtained by partitioning each image. Then, the set of all objects in the perceptual system is given by $Z = X \cup Y$. Objects in this system can be described by probe functions that operate on images. Simple examples include average colour, or maximum intensity (see, *e.g.*, [Henry and Peters (2009c); Marti *et al.* (2001)] for other examples of image probe functions).

An example of near images is given in Fig. 3 where Fig. 3(a) is being compared first to itself and then to Fig.'s 3(b)-3(e). Each image is a Bitmap of size 200×200, each coloured square has dimensions 100×100, and the size of each subimage is 10×10. The NMs were calculated using the average greyscale, denoted $\mathcal{B} = \{\phi_{\text{avg}}(f_s)\}$, for both the indiscernibility relation (Defn. 1) and the tolerance relation (Defn. 3) where $\varepsilon = 0.2$[b]. First, consider the results of the NM using Defn. 1. Since classes are formed with objects having matching descriptions, the results of the NM can be deduced intuitively. For example, Fig.'s 3(a) & 3(b) differ only by the lower left square. Thus, 75% of the objects in $Z = X \cup Y$ will have matching

[b]Note, the result of all probe function values are normalized to the interval $[0, 1]$, thus, in this case, selecting $\varepsilon = 0.2$ is equivalent to finding all objects whose greyscale value is within 51 (20% of 255) of each other.

descriptions. Conversly, determining the results of the NM using Defn. 3 is not quite as easy. Selecting an epsilon of 0.2 creates a tolerance class consiting of all subimages located in the top half of Fig. 3(a), *i*.e., the greyvalues of the top two squares differ by less than 51 (20% of 255). Using this informatin along with the data given in Table 2, the tolerance NM between Fig.'s 3(a) & 3(b) is calculated as

$$NM_{\cong_{\mathcal{B}}} = \frac{1}{800}\left(1 \cdot 400 + 1 \cdot 200\right) = 0.75,$$

and the NM between Fig.'s 3(a) & 3(c) is calculated as

$$NM_{\cong_{\mathcal{B}}} = \frac{1}{800}\left(0.5 \cdot 300 + 1 \cdot 200\right) = 0.4375.$$

Notice that the NM is lower using the tolerance relation when comparing Fig.'s 3(a) & 3(c). This is easiest explained by observing that selecting $\epsilon = 0.2$ and comparing Fig.'s 3(a) & 3(c) using the tolerance relation produces the same result as comparing Fig. 4 to Fig. 3(c) using the indiscernibility relation. Observe that both sets of images share the fact that their lefts sides are different, however, Fig.'s 4 & 3(c) are more dissimilar due to the lack of representation of the top half of Fig. 4 in Fig. 3(c). In other words, more information is lost by colouring the left side of Fig. 4 black than by colouring the left side of Fig. 3(a) because of the added loss of the information contained in the top half of Fig. 4 (a fact that is reflected in the NMs).

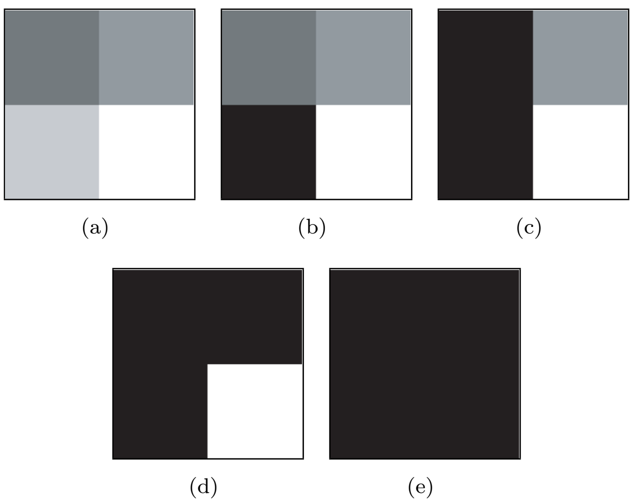
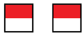
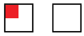
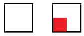
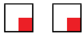
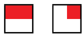
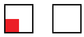
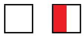
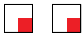


Fig. 3. Example of NM comparing first image to the remaining four: (a) Test pattern for comparison (note, $NM_{\sim_{\mathcal{B}}} = NM_{\cong_{\mathcal{B}}} = 1$ when compared to itself), (b) $NM_{\sim_{\mathcal{B}}} = NM_{\cong_{\mathcal{B}}} = 0.75$, (c) $NM_{\sim_{\mathcal{B}}} = 0.5$, $NM_{\cong_{\mathcal{B}}} = 0.4375$, (d) $NM_{\sim_{\mathcal{B}}} = NM_{\cong_{\mathcal{B}}} = 0.25$, and (e) $NM_{\sim_{\mathcal{B}}} = NM_{\cong_{\mathcal{B}}} = 0$..

Table 2. $NM_{\cong_{\mathcal{B}}}$ Calculation Example.

Images	Tolerance Class (TC)	TC Size	Object in X	Objects in Y	TC Ratio
		400	200	200	1
		100	100	0	0
		100	0	100	0
		200	100	100	1
		300	200	100	0.5
		100	100	0	0
		200	0	200	0
		200	100	100	1

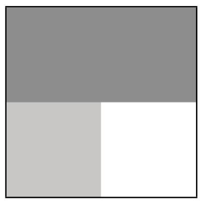


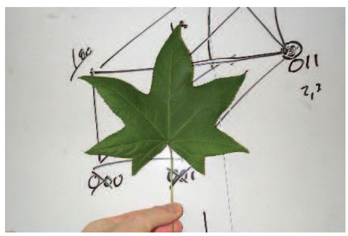
Fig. 4. Example showing similarity between $NM_{\sim_{\mathcal{B}}}$ and $NM_{\cong_{\mathcal{B}}}$: Comparing Fig.'s 3(a) & 3(c) using a tolerance relation and $\epsilon = 0.2$ is equivalent to comparing this figure with Fig. 3(c) using the indiscernibility relation.

Next is another example providing a visual representation of both equivalence and tolerance classes. Fig. 5 consists of images from the Berkeley Segmentation Dataset [Martin *et al.* (2001)] and the Leaves Dataset [Weber (1999)], and Fig. 6 consists of images depicting the equivalence and tolerance classes created from Fig 5(a). Fig. 6(a) is a visual representation of the elementary sets (see, *e.g.*, Eq. 1) obtained using $\mathcal{B} = \{\phi_{\text{avg}}(f_s)\}$, and created by the NEAR system (see [Henry and Peters (2009c)]). In other words, each grey level shown in the image is a label assigned to a particular elementary set, and the image is a visualization of the set of labels assigned to the objects (small pixel windows of size 5×5) obtained from Fig. 5. Notice, that objects from a given elementary set are not necessarily restricted to the same location within the image due to the creation of elementary sets based on object features (*i.e.*, the covering occurs in a feature space).

Next, Fig.'s 6(b)-6(d) provide visualizations of tolerance classes created from Fig. 5. Recall that the tolerance relation induces a covering of the objects instead of a partition (*i.e.* a single subimage can belong to more than one class). As a result, Fig.'s 6(b)-6(d) show the number of classes each subimage belongs to instead of the tolerance classes themselves. Fig. 6(b) uses greyscale values to represent the number of classes a subimage belongs to, where the grey levels white and black correspond to an object belonging to 255 and 0 classes respectively. The tolerance classes in these images were created using $\epsilon = 0.1$, a window size of 10 pixels, and $\mathcal{B} = \{\phi_{\text{avg}}(f_s)\}$. Notice, that the white (background) subimages belong to more tolerance classes than the green (leaf) subimages because the area of the leaf is much smaller than the background, *i*.e., each white subimage has more opportunity to be part of a tolerance class.

Finally, Fig. 7 is a plot of NM values comparing the nearness of Fig.'s 5(a) & 5(b) and Fig.'s 5(a) & 5(c) using the normalized green value from the RGB colour model and Pal's entropy, respectively denoted $\mathcal{B} = \{\phi_{\text{NormG}}(f_s), \phi_{\text{H}_{\text{Pal}}}(f_s)\}$ (see [Henry and Peters (2009c,d)]). Furthermore, the results were obtained using $\epsilon = 0, 0.01, 0.05, 0.1$ (note, the indiscernibility relation is used for $\epsilon = 0$), and a subimage size of 10×10. Observe that the two leaf images produce higher NMs than Fig. 5(a) and the Berkeley image because the leaf images produce objects that have more in common in terms of their descriptions (using the probe functions in $\mathcal{B}$). These results match our perception of the similarity between these three images. Lastly, note that the values using the indiscernibility relation are quite similar (near zero). In practise features values tend not to be exactly equal thus producing lower NMs. As shown by the results, this problem can be overcome by using the tolerance relation.

(a)



(b)

(c)

Fig. 5. Samples from image databases: (a), (b) Leaves Dataset [Weber (1999)], and (c) Berkeley Segmentation Dataset [Martin *et al.* (2001).]

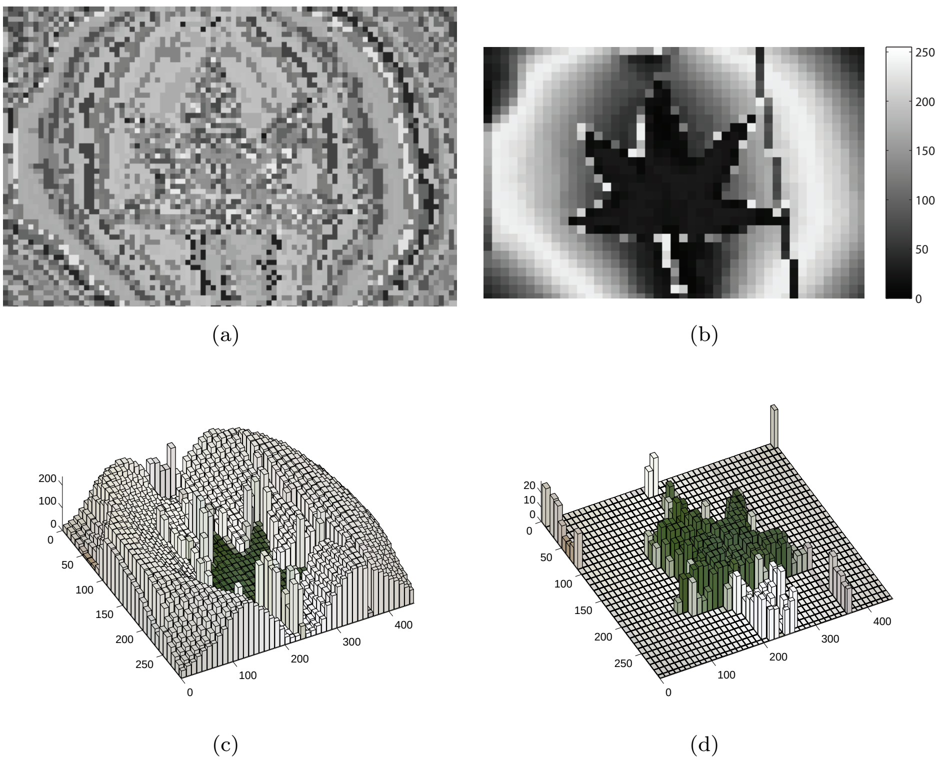


Fig. 6. Examples showing visualization of equivalence and tolerance classes obtained from image Fig 5(a): (a) Equivalence classes created using $\mathcal{B} = \{\phi_{\text{avg}}(f_s)\}$, (b) image showing tolerance class membership (dark regions correspond to low membership), (c) 3D plot of (b), and (d) plot demonstrating that the membership of objects belonging to the leaf is not zero (as appears to be the case in (b) and (c)).

4. Hausdorff Distance and Image Correspondence

The Hausdorff distance is used to measure the distance between sets in a metric space [Hausdorff (1914)] (see [Hausdorff (1962)] for English translation), and has been traditionally used in CBIR (see, *e.g.* [Smeulders *et al.* (2000); Park *et al.* (2008)]). The Hausdorff distance is defined as

$$d_H(X,Y) = \max\{\sup_{x\in X}\inf_{y\in Y} d(x,y), \sup_{y\in Y}\inf_{x\in X} d(x,y)\},$$

where sup and inf refer to the supremum and infimum, and $d(x,y)$ is the distance metric (in this case it is the l^2 norm). The Hausdorff distance is a natural choice for comparison with the NM, since it is a measure of the distance between sets.

The method of applying the Hausdorff distance to the image correspondence problem is the same as that described in Section 3. To reiterate, consider Fig. 8 where each rectangle represents a set of subimages (obtained by partitioning the

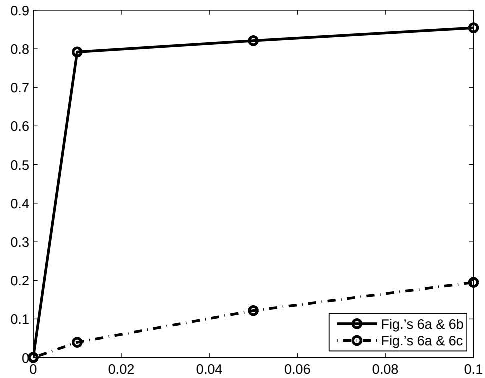


Fig. 7. Plot showing NM values comparing Fig.'s 5(a) & 5(b) and Fig.'s 5(a) & 5(c) for $\varepsilon = 0, 0.01, 0.05, 0.1$

original images X and Y) and the coloured areas represent some of the obtained tolerance classes[c]. Note, the tolerance classes are created based on the feature values of the subimages, and consequently, do not need to be situated geographically near each other (as shown in Fig. 8). In the case of the NM, the idea is that similar images should produce tolerance classes with similar cardinalities. Consequently, we are comparing the cardinalities of the portion of a tolerance class belonging to set X with the portion of the tolerance class belonging to set Y (represented in Fig. 8 as sets with the same colour). In contrast, the Hausdorff distance measures the distance between sets in some metric space. As a result, we measure distance in the feature space between the portion of a tolerance class belonging to set X with the portion of a tolerance class belonging to set Y (again, represented as a sets with the same colour). Here, the idea is that images that are similar should have tolerance classes are the close (in the Hausdorff sense) in the feature space. As a result, low Hausdorff distances are desirable.

More recently, a new form of the Hausdorff distance, called the Perceptually Modified Hasudorff Distance (PMHD), has been introduced that is in-line with the notion of the perceptual underpinnings of near set theory. The PMHD is "perceptual" in the in the sense that humans view/recall an image by its dominant colours and the proportions of dominate colours [Park *et al.* (2008)]. Consequently, the PMHD is defined as follows. First, partition the images X and Y based on their dominate colours. In this case we used the same approach given in [Park *et al.* (2008)], namely, the images were partitioned the image using the k-means clustering algorithm with $k = 10, 30$ in the CIELab colour space. Once the images have

[c]The tolerance relation covers both images, but not all the classes are shown in the interest of clarity

been partitioned into regions based on dominate colours, a statistical signature of the images can be defined as

$$\mathcal{S} = \{(\boldsymbol{s}_i, w_i, \boldsymbol{\Sigma}_i) \mid i = 1, \ldots, N\},$$

where N is the number of regions (clusters), $\boldsymbol{s}_i$ is the mean feature vector (mean CIELab colour) of region i, and w_i is the number of vectors (in this case pixels) that belong to region i, and Σ_i is the covariance matrix of the i^{th} cluster. Next, given two statistical signatures

$$\mathcal{S}_1 = \{(\boldsymbol{s}_i^1, w_i^1, \boldsymbol{\Sigma}_i^1) \mid i = 1, \ldots, N\} \text{ and}$$
$$\mathcal{S}_2 = \{(\boldsymbol{s}_j^2, w_j^2, \boldsymbol{\Sigma}_j^2) \mid j = 1, \ldots, N\},$$

the PMHD is defined as

$$d_{PMH}(\mathcal{S}_1, \mathcal{S}_2) = \max\{d_{PM}(\mathcal{S}_1, \mathcal{S}_2), d_{PM}((\mathcal{S}_2, \mathcal{S}_1)\},$$

where

$$d_{PM}(\mathcal{S}_1, \mathcal{S}_2) = \frac{\sum_{i=1}^{N}[w_i^1 \times \min_j(d(\boldsymbol{s}_i^1, \boldsymbol{s}_j^2)/\min(w_i^1, w_j^2))]}{\sum_{i=1}^{N} w_i^1}.$$

Here $d(\boldsymbol{s}_i^1, \boldsymbol{s}_j^2)$ is the Euclidean distance between the two vectors.

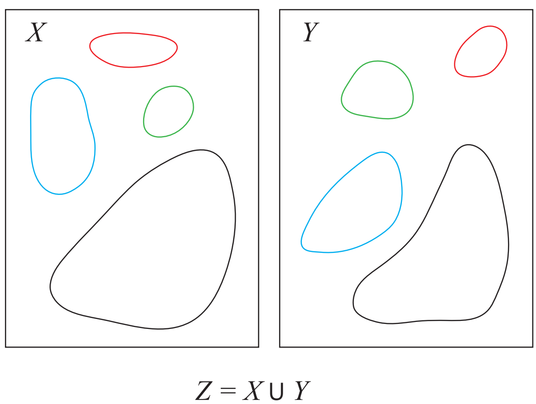


Fig. 8. Graphical representation of image correspondence problem.

5. Results

This section presents results of performing CBIR using the NM, and both forms of the Hausdorff distance. The plot given in Fig. 7 suggests that the NM would be useful in measuring the similarity of images. To investigate this property further, we used the Berkeley Segmentation Dataset [Martin *et al.* (2001)] and the Leaves

Dataset [Weber (1999)] (both freely available online) to perform CBIR using the NM, and both Hausdorff distances. Specifically, the image in Fig. 5(a) was selected as the query image and is compared to 200 images (100 from both the leaves and Berkeley datasets, respectively)[d], where the idea result of any of the measures would be to have a higher value for all 100 leaf images since the query image belongs to the leaves dataset. In other words, we hope to see the lowest NM value for the leaf images be higher than the highest Berkeley NM value. Note, this would be the opposite for the Hausdorff measures, as lower Hausdorff values correspond to a higher degree of similarity. Also, as above, the probe functions used to generate the tolerance classes were $\mathcal{B} = \{\phi_{\text{NormG}}(f_s), \phi_{\text{H}_{\text{Pal}}}(f_s)\}$, and the window size was 10×10. Finally, the measures were compared using precision versus recall plots. Precision/recall plots are the common metric for evaluating CBIR systems where precision and recall are defined as

$$\text{precision} = \frac{|\{\text{relevant images}\} \cap \{\text{retrieved images}\}|}{|\{\text{retrieved images}\}},$$

and

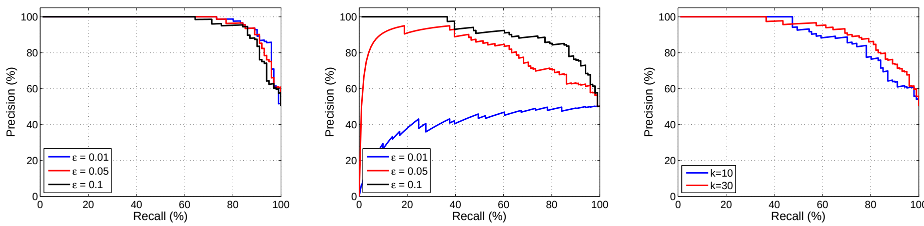
$$\text{recall} = \frac{|\{\text{relevant images}\} \cap \{\text{retrieved images}\}|}{|\{\text{relevant images}\}}.$$

In the idea case (described above), precision would be 100% until recall reached 100%, at which point precision would drop to # of images in query category / # of images in the database. In our case, the final value of precision will be 50% since there are two categories each containing 100 images.

The results of these comparisons are given in Fig. 9 & 10, where Fig. 9 contains the results of the queries for each individual measure, and Fig. 10 is a comparison of the best results from each measure. The "best" results were selected by choosing the parameters that returned the most leaf images before a Berkeley image was selected. Notice that the best query results for the NM occur for $\epsilon = 0.1$, the best results for original Hausdorff measure occur for $\epsilon = 0.1$, and the best results for the PMHD occur for $k = 10$. Furthermore, notice, as given in Fig. 10, that the NM produces the best precision/recall plot with 73 images retrieved from the leaves dataset before a Berkeley image is selected, while both Hausdorff measures return 37 leaf images before a Berkeley image is selected. Note, while the PMHD is based on the original Hausdorff measure, the fact that they both retrieved 37 images is a coincidence due to the different nature of the underlying data being measured. Recall, that the original Hausdorff measure is being used to measure the distance between the portion of a tolerance class belong to image X and the portion of the tolerance class belong to image Y, while the PMHD is comparing signatures of clusters obtained by partitioning the images using the k-means clustering algorithm. However, an important observation can be made given that the Hausdorff distance

[d]Note, the number of pixels in the leaf images were decimated by a factor of 4 to be closer in size to the Berkeley images, *i.e.*, their dimension was reduced from 896×592 to 448×296.

is producing results comparable to the PMHD. Namely, that it appears as though measuring the Hausdorff distance between portions of tolerance classes contained in the two images produces better results than using it to compare sets of points in colour space as is the usual method as evidenced by the similar results of the two Hausdorff. However, experimental results of the two methods are needed before any claim can be made.



(a) (b) (c)

Fig. 9. Results of CBIR using Fig. 5(a) as query image: (a), (b) respective results of NM and HD for $\epsilon = 0.01, 0.05, 0.1$, and (c) results of PMHD with $k = 10, 30$.

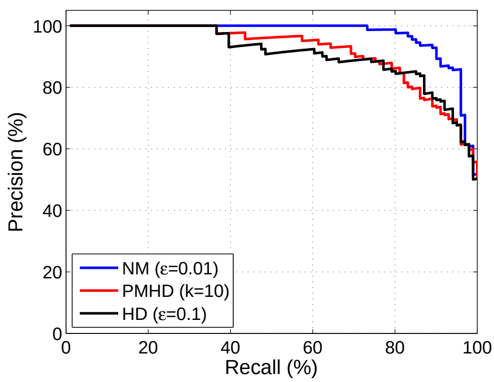


Fig. 10. Comparison of best results of NM, HD, and PMHD from Fig. 9.

What we can say for certain is that, based on these results, the NM outperforms the PMHD suggesting that the perceptual information contained in tolerance classes provides better retrieval results than comparison of the dominant image colours of the PMHD. These results match intuition in that, at some level, our mind assesses similarity by comparing the descriptions of the objects we are considering, and that the comparison is not based on exact values (*i.e.*, the equivalence of features) but rather our mind easily allows some tolerance in making these comparisons.

Consequently, it makes sense that a measure modelled on these types of comparisons would produce the best results.

6. Conclusion

This article presents a practical application of near sets in discovering similar images and in measuring the degree of similarity between images. Near sets themselves reflect human perception, *i.e.*, emulating how humans go about perceiving and, possibly, recognizing objects in the environment. Although a consideration of human perception itself is outside the scope of this article, it should be noted that a rather common sense view of perception underlies the basic understanding of near sets (in effect, perceiving means identifying objects with common descriptions). And perception itself can be understood in Maurice Merleau-Ponty's sense [1945], where perceptual objects are those objects captured by the senses. In presenting this application, this article has given details on how to apply near set theory to the problem of images correspondence by way of calculating the nearness of images. The results presented here demonstrate that the NM measure can be used effectively to create CBIR systems. Moreover, it is the case that the choice of probe functions is very important. The results obtained so far in comparing nearness measures and Hausdorff distance are promising. Future work in this research includes further comparisons between the Hausdorff distances and NMs, and comparison of the original Hausdorff distance on set of image points with that of the Hausdorff distance applied to tolerance class (as was done in this article). Also, future work will involve refining the NM to allow it to be used for indexing. For instance, the NM in its current form must be calculated anew for each image query. However, it should be possible to develop a measure that can be calculated for each image in the database such that each new query only needs to be compared with pre-determined NM values and not recalculated each time. What is certain is that the results presented in this article demonstrate that near set theory can be a useful tool in image correspondence systems, and that perception-based image retrieval is possible.

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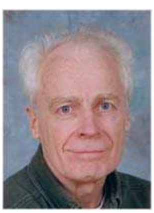
Biography

Christopher Henry received his M.Sc. degree in 2006 from the University of Manitoba. Currently, he is a Ph.D. candidate in the Department of Electrical and Computer Engineering (ECE) at the University of Manitoba. Since 2004, he has published fourteen articles (six journal publications, six conference publications, and two book chapter) on near sets, content based image retrieval, reinforcement learning, rough sets, intelligent systems, and pattern recognition. Similarly, he has presented papers at two international conferences. His current research interests are in content based image retrieval, near sets, reinforcement learning, rough set theory, approximation spaces, digital image processing, pattern recognition, and intelligent systems.



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